Parameterizing PI Congestion Controllers

Ahmad T. Al-Hammouri, Vincenzo Liberatore, Michael S. Branicky
Case Western Reserve University

Stephen M. Phillips
Arizona State University

April 3, 2006

Support by: NSF CCR-0329910, Department of Commerce TOP 39-60-04003, NASA NNC04AA12A, and an OhioICE Training grant

Contributions of the Paper

- Complete stability region for PI
- Presents examples that show
  - Different stable PI parameters exhibit widely different control performance
  - Neglecting delays in control design leads to unstable systems
TCP-AQM Control Loop

TCP source → Router ← TCP sink

Plant

\[ P(s) = \frac{q(t)}{s(t - cwnd/RTT)} \]

On Ack
\[ cwnd += 1/\text{cwnd} \]

On loss
\[ cwnd /= 2 \]

Controller

\[ G(s) = \frac{k_p + k_i}{s} \]

On Ack
\[ q(t) = \Sigma x(t) - C \]

On loss
\[ q(t) = 0 \]

On Ack
\[ q(t) = B \]

On loss
\[ q(t) = q_0 \]

Integral term eliminates the steady-state error

\[ u(t) = k_p \cdot e(t) + k_i \int_0^t e(\tau) d\tau \]

\[ G(s) = k_p + k_i \]

\[ P(s) = \frac{q(t)}{s(t - cwnd/RTT)} \]

\[ q(t) = \Sigma x(t) - C \]

\[ q(t) = 0 \]

\[ q(t) = B \]

\[ q(t) = q_0 \]

PI AQM [Holot et al 2001]

- PI vs RED
- Control signal:
  - \[ u(t) = k_p \cdot e(t) + k_i \int_0^t e(\tau) d\tau \]
- Frequency transfer fn:
  - \[ G(s) = k_p + k_i \]
  - \[ P(s) = \frac{q(t)}{s(t - cwnd/RTT)} \]
  - \[ q(t) = \Sigma x(t) - C \]
  - \[ q(t) = 0 \]
  - \[ q(t) = B \]
  - \[ q(t) = q_0 \]
Parameterizing PI AQM

- Problem:
  - Determine the **entire region** of stabilizing $k_p$ and $k_i$ values

- Objectives:
  - Stable closed-loop system
  - Enhanced closed-loop performance
    - Steady-state error, convergence time, overshoot

- Challenges:
  - Delays

Contributions of the Paper

- **Complete stability region for PI**
  - Presents examples that show
    - Different stable PI parameters exhibit widely different control performance
    - Neglecting delays in control design leads to unstable systems
Complete Stability Region $S_R$ [Silva et al 2005]

- $S_R = (S_0 \setminus S_N) \setminus S_L$
- $S_0$:
  - Stability region for the delay-free system
  - $P_0(s) = \frac{B}{(s + \alpha)(s + \beta)}$
- $S_N$:
  - $\{(k_p, k_i)\}$:
  $$\lim_{s \to \infty} \left| \frac{G(s)P(s)}{s(s + \alpha)(s + \beta)} \right| B \leq 1$$

Determination of $S_L$

- Set of $k_p$ and $k_i$’s that destabilizes the closed-loop for delays less than $d_0$
- For given $k_p$ and $k_i$
  - Find $d$ that gives the blue curve
  - If $(d \leq d_0)$, $(k_p, k_i) \in S_L$
  - Else, $(k_p, k_i) \notin S_L$
- Sweep $\forall (k_p, k_i) \in S_0$
Complete Stability Region $S_R$

- $S_R = (S_0 \setminus S_N) \setminus S_L$

Contributions of the Paper

- Complete stability region for PI
- Presents examples that show
  - Different stable PI parameters exhibit widely different control performance
  - Neglecting delays in control design leads to unstable systems
Example 1

- $N = 75; \ d_0 = 0.15 \text{ sec}; \ C = 1250 \text{ pkt/sec}$

Contributions of the Paper

- Complete stability region for PI
- Presents examples that show
  - Different stable PI parameters exhibit widely different control performance
  - **Neglecting delays in control design leads to unstable systems**
**Example 2 [Heying et al 2002]**

- \( N = 60; \ d_0 = 0.22 \text{ sec}; \ C = 1250 \text{ pkt/sec} \)

**PIP Controller [Heying et al 2002]**

\[
q(s) \quad e \quad G(s) \quad u \quad P(s) \quad q'(s)
\]

\[
q_0 + _e \quad u \quad P(s) \quad q'(s)
\]

\[
G(s) \quad P(s) \quad q'(s)
\]

\[
q(s) \quad e \quad G(s) \quad u \quad P(s) \quad q'(s)
\]

\[
K_h
\]

\[
K_h
\]

\[
K_h
\]

\[
K_h
\]

Ahmad Al-Hammouri
Parameterizing PI Congestion Controllers
Feb/BD’06

Ahmad Al-Hammouri
Parameterizing PI Congestion Controllers
Feb/BD’06
Future Work

- Conduct packet-level simulations (ns-2)
- Define a “Networks Performance” objective function
  - Optimize the objective function over the stability region
- Analyze the queue nonlinearity (i.e. truncation)

Thank You

- Questions
- Comments