Equivalence of Analog and Digital Computation

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Abstract

We discuss a class of ordinary differential equations (ODEs) that serve as a model for analog computation. We then show that analog computation in this model is equivalent to traditional discrete computation. In particular, we produce ODEs in this class which simulate arbitrary finite automata (FA) and Turing machines.

The paper is organized as follows: We start with a discussion about models of computation and simulation. We then give the rudiments of an ODE-based model of analog computation. The stage is thus set for solving two problems of simulating discrete computation via our analog model.

Firstly, we give a set of differential equations that simulate arbitrary finite automata. Each step of the FA's computation is explicitly simulated by this set of differential equations.

Secondly, we give a set of differential equations that robustly simulate an arbitrary Turing machine (TM) given a (nonexact) timing pulse. Again, each step of the TM's computation is simulated by our set of ODEs. By simulating a universal TM, we have a set of ODEs which are universal computers at least as powerful as Turing machines. Conversely, one can use TMs to simulate these ODEs. We use results in the literature to show that ODEs of this form are efficiently simulated by TMs (polynomial time per step). Thus, we exhibit the polynomial time complexity equivalence of analog and digital computation.

Finally, we discuss elements that a more comprehensive ODE-based model of analog computation should contain.